# HYDRAULIC TURBOMACHINES
# Mock exam – part 2

Duration : 2 hours;

Documentation: personal hand written notes, bilingual dictionary, and available lectures and exercises on the Moodle are authorized; you can use your laptop in offline mode (no internet connection).

Exam evaluation: The weight of each question is indicated. General presentation and clarity of answers will be taken into account for the evaluation.

Maximum total score: 30 points

**Pumped storage power plant**

The outline of a pumped storage power plant is shown in Figure 1. The power plant features 2 units, each one equipped with a pump-turbine. Answer to the questions based on the values provided in Table 1. The gravity acceleration, the water density and water kinematic viscosity are $g = 9.81$ m s$^{-2}$, $\rho = 998$ kg m$^{-3}$ and $\nu_{water} = 10^{-6}$ m$^2$ s$^{-1}$. In Table 1, the main operating parameters of the power plant are listed.

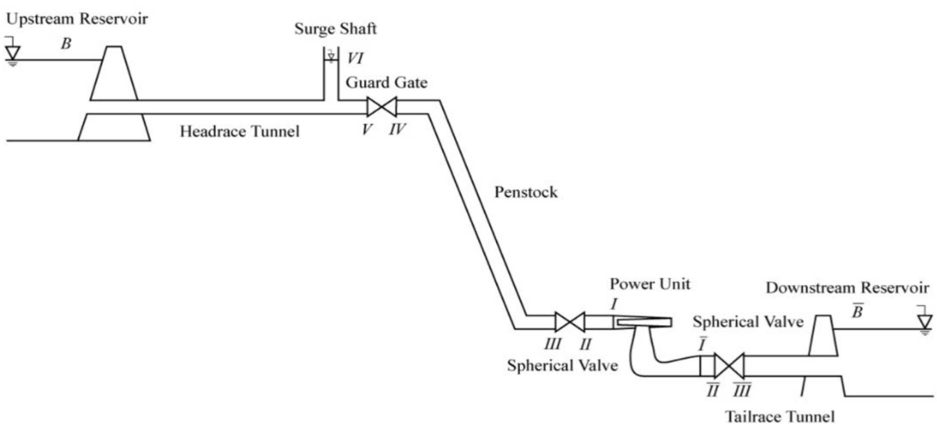


*Figure 1 – Outline of the power station*

*Table 1 – Fixed parameters of the power plant*

| **Data** | ***Symbol*** | ***Value*** | ***Unit*** |
|---|---|---|---|
| Atmospheric pressure | $p_a$ | $1\times10^5$ | Pa |
| Saturated vapor pressure | $p_v$ | 2343 | Pa |
| Headwater reservoir level | $Z_B$ | 1250 | m |
| Tailwater reservoir level | $Z_{\overline{B}}$ | 1025 | m |
| Amount of units in the power plant | $z_{machines}$ | 2 | - |
| Number of poles pairs | $z_{p,pairs}$ | 10 | - |

| | | | |
|---|---|---|---|
| Frequency of the grid | $f_{grid}$ | 50 | Hz |
| Inlet (in turbine mode) section diameter | $D_{1e}$ | 3.50 | m |
| Inlet (in turbine mode) section height | $B$ | 0.60 | m |
| Outlet (in turbine mode) section diameter | $D_{\bar{1}e}$ | 2.80 | m |

**Pumped storage power plant – turbine mode**

*Table 2 – Turbine mode parameters of the power unit*

| | | | |
|---|---|---|---|
| Energetic efficiency | $\eta_e$ | 0.92 | - |
| Volumetric efficiency | $\eta_q$ | 0.99 | - |
| Global machine efficiency | $\eta$ | 0.90 | - |
| Rated discharge per unit | $Q$ | 50 | $m^3 s^{-1}$ |

At first, we consider the power plant in generating mode. The operation parameters are listed in Table 2.

1 Compute the specific potential energy of the installation. [1 point]

$$gH_B - gH_{\bar{B}} = g(Z_B - Z_{\bar{B}}) = 2207.2 \text{ Jkg}^{-1}$$

2 For the rated discharge, the head losses of the installation have been measured and are equal to $\sum gH_r$ = 201.4 J kg$^{-1}$. Compute the available specific energy of the turbines. Deduce the net head $H$ of the turbines.

$$E = gH_B - gH_{\bar{B}} - \sum gH_r = 2005.8 \text{ J kg}^{-1}$$

$$H = \frac{E}{g} = 204.5 \text{ m}$$

3 Compute the runner frequency $n$ (Hz) and the specific speed $\nu$ of the runner. [1 point]

*The runner frequency is defined by the grid frequency and the amount of pole pairs of the electrical machine:*

$$n = \frac{f_{grid}}{z_{p,pairs}} = 5 \text{ Hz}$$

$$\omega = 2\pi n = 31.42 \text{ rad s}^{-1}$$

$$\nu = \frac{\omega\sqrt{Q}}{\sqrt{\pi}(2E)^{\frac{3}{4}}} = 0.249$$

4 Compute $P_h$, the hydraulic power. [1 point]

$$P_h = \rho\, Q\, E = 100.09 \text{ MW}$$

5 Compute the transferred specific energy $E_t$. [1 point]

$$E_t = \eta_e E = 1845.3 \text{ J kg}^{-1}$$

6 Compute the torque experienced by the runner shaft $T_t$. [1 point]

*We need to consider the leakage flow loss as well:*

$$P_t = \omega T_t = \rho\, Q_t\, E_t = \rho\, Q\, \eta_q\, E_t = 91.16 \text{ MW}$$

$$T_t = \frac{P_t}{\omega} = \frac{91.16 \text{ MW}}{31.42 \text{ rad}\cdot\text{s}^{-1}} = 2.9\times 10^6 \text{ Nm}$$

7 Compute the mechanical efficiency ($\eta_{me} = \eta_{rm}\cdot\eta_m$). [1 point]

$$\eta_{me} = \frac{P}{P_t} = \frac{P_h\cdot\eta}{P_t} = \frac{100.09\cdot 0.9}{91.16} = 0.988$$

8 Express the Net Positive Suction Specific Energy (NPSE) by $gH_{\bar{I}}$, the machine altitude level $Z_{ref}$, and the saturated pressure $p_v$. [1 point]

$$NPSE = gH_{\bar{I}} - \frac{p_v}{\rho} - gZ_{ref}$$

9 Express Thoma number $\sigma = \frac{NPSE}{E}$, using the setting level $h_s = Z_{ref} - Z_{\bar{B}}$, the flow velocity $C_{\bar{I}}$, the available specific energy $E$, the saturated pressure $p_v$ and the atmospheric pressure $p_a$. Neglect singular and regular losses inside the tailrace tunnel, and assume that the tailrace tunnel outlet behaves as a water outflow (i.e. a sudden exit). [2 point]

*Neglect losses in tailrace tunnel + tunnel outlet as sudden exit:*

$$gH_{\bar{I}:-\bar{B}} = \frac{C_{\bar{I}}^{\,2}}{2} \rightarrow gH_{\bar{I}} = \frac{p_{\bar{B}}}{\rho} + gZ_{\bar{B}} + \frac{C_{\bar{I}}^2}{2}$$

*The Thoma number can be expressed as:*

$$\sigma = \frac{NPSE}{E} = \frac{\frac{p_a}{\rho} + gZ_{\bar{B}} + \frac{C_{\bar{I}}^2}{2} - \frac{p_v}{\rho} - gZ_{ref}}{E} = \frac{\frac{p_a - p_v}{\rho} - gh_s + \frac{C_{\bar{I}}^2}{2}}{E}$$

10 Assuming a draft tube outlet section of 25 $m^2$, compute the required $Z_{ref}$ to achieve a Net Positive Suction Head (NPSH) of 11.9 m. [2 point]

*This condition can be expressed as:*

$$NPSH = H_{\bar{I}} - \frac{p_v}{g\rho} - Z_{ref} = 11.9m$$

*The flow velocity at the draft tube outlet:* $C_{\bar{I}} = \frac{Q_t}{A_{\bar{I}}} = \frac{0.99\cdot 50m^3\cdot s^{-1}}{25m^2} = 1.98m\cdot s^{-1}$

*Therefore, isolating $H_{\bar{I}}$ from the expression found in question 9:*

$$Z_{ref} = \frac{p_a - p_v}{\rho g} + Z_{\bar{B}} + \frac{C_{\bar{I}}^2}{2g} - NPSH = \frac{1\times 10^5 - 2343}{998\cdot 9.81} + 1025 + \frac{1.98^2}{2\cdot 9.81} - 11.9 = 1023m$$

**Pumped storage power plant – pump mode**

*Table 3 – Pump mode parameters of the power unit*

| | | | |
|---|---|---|---|
| Energetic efficiency | $\eta_e$ | 0.92 | - |
| Volumetric efficiency | $\eta_q$ | 0.99 | - |
| Global machine efficiency | $\eta$ | 0.90 | - |
| Rated discharge per unit | $Q$ | 48 | $m^3s^{-1}$ |

Let's now move to study the power plant in pumping mode. The operation parameters are listed in Table 3. For this operating condition, it is assumed that the inlet velocity $\vec{C}_{\bar{1}}$ is axial and uniformly distributed. The outlet flow is also radially uniform. All the flow distribution coefficients $k_{cu}$ and $k_{cm}$ of global Euler equation, are then assumed equal to 1.

11 Compute the meridional component of the inlet absolute velocity $Cm_{\bar{1}e}$. [1 point]

*To compute the meridional component of the absolute velocity at the pump inlet, we need to consider the discharge before the leakage losses:*

$$Cm_{\bar{1}e} = \frac{Q_t}{A_{\bar{1}e}} = \frac{4 \cdot Q}{\pi D_{\bar{1}e}^{2} \cdot \eta_q} = \frac{4 \cdot 48}{\pi 2.8^2 \cdot 0.99} = 7.87 m \cdot s^{-1}$$

12 Compute the inlet tangential velocity $U_{\bar{1}e}$ and the relative inlet flow angle $\beta_{\bar{1}e}$. [1 point]

*The rotational speed $\omega$ in pump mode is the same as in turbine mode since the grid frequency doesn't change:* $U_{\bar{1}e} = \frac{\omega \cdot D_{\bar{1}e}}{2} = \frac{31.42 \cdot 2.8}{2} = 43.99 m \cdot s^{-1}$

*And* $\tan\left(\beta_{\bar{1}e}\right) = \frac{Cm_{\bar{1}e}}{U_{\bar{1}e}} \rightarrow \beta_{\bar{1}e} = \tan^{-1}\left(\frac{Cm_{\bar{1}e}}{U_{\bar{1}e}}\right) = 10.14^\circ$

13 The hydraulic power in pump mode for this power plant corresponds to the 97% of the hydraulic power in turbine mode. Compute the specific energy. [2 point]

*In turbine mode:* $P_h = 100.09 \text{ MW}$

*Therefore, in pump mode:* $P_h = 100.09 \text{ MW} \cdot 0.97 = 97.09 \text{ MW}$

*The specific energy reads :* $E = \frac{P_h}{\rho \cdot Q} = \frac{97.09}{998 \cdot 48} = 2026.8 \text{ J} \cdot \text{kg}^{-1}$

14 Compute the transferred specific energy. [1 point]

*The transferred specific energy reads :* $E_t = \frac{E}{\eta_e} = \frac{2026.8 \text{ J} \cdot \text{kg}^{-1}}{0.92} = 2203.0 \text{ J} \cdot \text{kg}^{-1}$

15 Compute the outlet tangential velocity $U_{1e}$ and the tangential component of the absolute outlet velocity $Cu_{1e}$. [1.5 point]

*Using Euler equation :* $E_t = Cu_{1e}U_{1e} - Cu_{\bar{1}e}U_{\bar{1}e} = Cu_{1e}U_{1e}$ *since the inlet flow is axial, i.e.* $Cu_{\bar{1}_e} = 0$

$$U_{1e} = \frac{\omega \cdot D_{1e}}{2} = \frac{31.42 \cdot 3.5}{2} = 54.99 m \cdot s^{-1} \text{ and } Cu_{1e} = \frac{E_t}{U_{1e}} = \frac{2203.0}{54.99} = 40.06 m \cdot s^{-1}$$

16 Is this operating condition the one corresponding to the maximum power transfer condition? Justify your answer. [1.5 point]

*No, the maximum power transfer condition is fulfilled when* $Cu_{1e} = \frac{U_{1e}}{2}$.

*In the current operating condition, we have* $Cu_{1e} = 40.06 m \cdot s^{-1} > \frac{U_{1e}}{2}$

17 Assume an outlet blade angle $\beta_{1b} = 24^{\circ}$ and no-slip condition fulfilled. For the same machine rotating velocity and the same efficiencies as listed in Table 3, provide the values of discharge $Q$ and net head $H$ that would allow the machine to operate in its maximum power transfer. [2.5 point]

*The pump can operate at the maximum power if* $Cu_{1e} = \frac{U_{1e}}{2} = \frac{\omega \cdot D_{1e}}{4}$. *For the same rotating velocity:* $Cu_{1e} = \frac{U_{1e}}{2} = \frac{54.99 m \cdot s^{-1}}{2} = 27.50 m \cdot s^{-1}$

*From Euler equation:* $E_t = Cu_{1e} \cdot U_{1e} = \frac{U_{1e}^2}{2}$ *and since* $E_t = g\frac{H}{\eta_e}$ *the net head H can be deduced:*

$H = \frac{E_t \cdot \eta_e}{g} = \frac{U_{1e}^2 \cdot \eta_e}{2 \cdot g} = \frac{54.99^2 \cdot 0.92}{2 \cdot 9.81} = 141.79 m$, *which corresponds to a specific energy of* $E = gH = 141.79 \cdot 9.81 = 1391 \text{ J} \cdot \text{kg}^{-1}$

*Moreover, at the maximum power condition, the following relation on* $Q_t$ *holds:*

$$Q_t = \frac{U_{1e}A_{1e}\text{tg}(\beta_{1e})}{2}.$$

*Assuming no-slip condition:*

$$\beta_1 = \beta_{1b} \rightarrow Q_t = \frac{U_{1e}A_{1e}\text{tg}(\beta_{1e})}{2} = \frac{U_{1e} \cdot \pi \cdot D_{1e} \cdot B \cdot \text{tg}(\beta_{1b})}{2} = \frac{54.99 \cdot \pi \cdot 3.5 \cdot 0.6 \cdot \text{tg}(24^{\circ})}{2} = 80.76 m^3 \cdot s^{-1}$$

*And the optimal discharge can be deduced* $Q = Q_t \cdot \eta_q = 80.76 \cdot 0.99 = 79.95 m^3 \cdot s^{-1}$

Now imagine that you are in charge of testing the reduced scale model of this pump-turbine in pumping mode, with a scale ratio $\lambda_{scale} = 4.80$. To ensure that the results you get from the tests are reliably representing the behavior of the real scale machine, you must correctly determine several operational parameters. To do that, consider the rated operating condition (and not the optimum computed in question 18) ). The subscript $_M$ indicates a quantity at Model scale, whereas the subscript $_P$ indicates the real scale machine (i.e. the Prototype scale).

18 Compute the different IEC Factors for speed, discharge, torque and power, considering 1% of bearing power losses. Use $D_{1e}$ as the reference diameter. [2 point]

*With data of Table 1, the rotational speed* $\omega = n \cdot 2\pi = -31.42 rad \cdot s^{-1}$ *and the torque computed as* $T_m = \frac{P_h \cdot \eta_m}{\eta \cdot \omega} = \frac{\rho \cdot Q \cdot E \cdot \eta_m}{\eta \cdot \omega} = \frac{998 \cdot 48 \cdot 2026.8 \cdot 0.99}{0.9 \cdot 31.42} = 3.4 \times 10^6\, N\,m$*:*

$$n_{ED} = \frac{nD_{1e}}{\sqrt{E}} = -0.3887 \quad Q_{ED} = \frac{Q}{D_{1e}^{\,2}\sqrt{E}} = -0.0870 \quad T_{ED} = \frac{T_m}{\rho D_{1e}^{\,3} E} = 0.0392$$

$$P_{ED} = \frac{P_m}{\rho D_{1e}^{\,2} E^{1.5}} = -0.0958 \text{ with } P_m = \omega T_m$$

19 Compute the pump inlet and outlet diameters of the reduced scale model $D_{\bar{1}e,M}$ and $D_{1e,M}$. [2 point]

*With a scale ratio* $\lambda_{scale} = 4.8$ *the diameters are scaled simply by:*

$$D_{\bar{1}e,M} = \frac{D_{\bar{1}e,P}}{\lambda_{scale}} = 0.583m \text{ and } D_{1e,M} = \frac{D_{1e,P}}{\lambda_{scale}} = 0.729m$$

20 Determine the net head at reduced model scale $H_M$ considering similarity of the Froude number between real scale and model scale. [1.5 point]

*Froude similarity:* $Fr_M = Fr_P$

$$Fr \sim \sqrt{\frac{H}{D}} \rightarrow H_M = \frac{H_P}{\lambda_{scale}} = \frac{2026.8}{9.81 \cdot 48} = 43.04m$$

21 Considering similarity of the IEC factors, compute the model scale rotational speed $\omega_M$, discharge $Q_M$ and mechanical torque $T_{m,M}$. [2 point]

*Similarity of IEC factors:* $n_{ED,M} = n_{ED,P}$, $Q_{ED,P} = Q_{ED,M}$ *and* $T_{ED,M} = T_{ED,P}$

*Therefore, from question 19):*

$$\omega_M = n_M \cdot 2\pi = \frac{-0.3887 \cdot \sqrt{E_M}}{D_{1e,M}} \cdot 2\pi = -68.84\, rad \cdot s^{-1}$$

$Q_M = -0.087 \cdot \sqrt{E_M} \cdot D_{1e,M}^{\,2} = -0.95 m^3 \cdot s^{-1}$ *(negative sign because in pump mode)*

$$T_M = 0.0392 \cdot \rho \cdot E_M \cdot D_{1e,M}^{\,3} = 6399 Nm$$

*Mechanical power at model scale:* $P_M = T_M \cdot \omega_M = -440.6\, kW$ *(same if computed starting from similarity of* $P_{ED}$ *factor)*